Work and Energy II

1. A particle of mass $m = 4\ kg$ moves from the origin to the position having coordinates $x = 5\ m$ and $y = 5\ m$ under the influence of gravity acting in the negative y direction (see Fig.1). Calculate the work done by gravity in going from O to C along (a) OAC, (b) OBC, (c) OC. Yours results should be all identical. Why?

2. A single conservative force acting on a particle varies as $\mathbf{F} = (-Ax + Bx^2)\hat{\imath}\ N$, where A and B are constants and x is in meters. (a) Calculate the potential energy associated with this force, taking $U = 0$ at $x = 0$. (b) Find the change in potential energy and change in kinetic energy as the particle moves from $x = 2.0\ m$ to $x = 3.0\ m$.

3. Show that any force of the form $\mathbf{F}(r) = F(r)\hat{\boldsymbol{\theta}}$, where $\hat{\boldsymbol{\theta}}$ is a unit vector perpendicular to the radius vector, is not a conservative force. In plane polar coordinates an infinitesimal displacement is $d\mathbf{s} = dr\hat{\mathbf{r}} + rd\theta\hat{\boldsymbol{\theta}}$.

4. A particle of mass m is released at height H and than slides on a frictionless surface that becomes a vertical circle of radius R, as shown in Fig.2. What is the minimum value of H for the particle not to lose contact at a height $h = \frac{3}{2}R$?

5. An Eskimo child slides on a icy (frictionless) hemispherical igloo of radius R, as shown in Fig.3. She starts with a negligible speed at the top. (a) At what angle to the vertical does she lose contact with the surface? (b) If there were the friction, would contact be lost at higher or lower point?

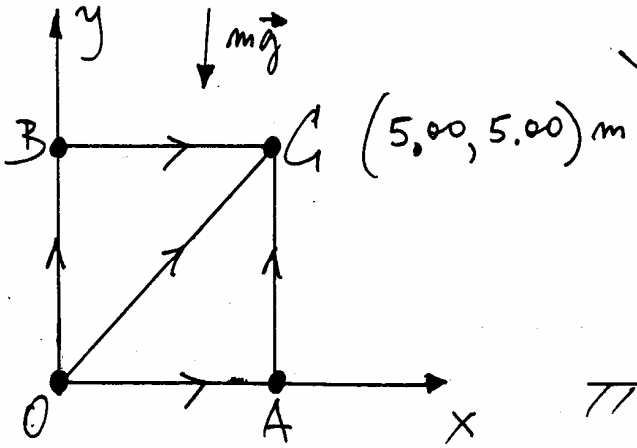

Fig. 1.

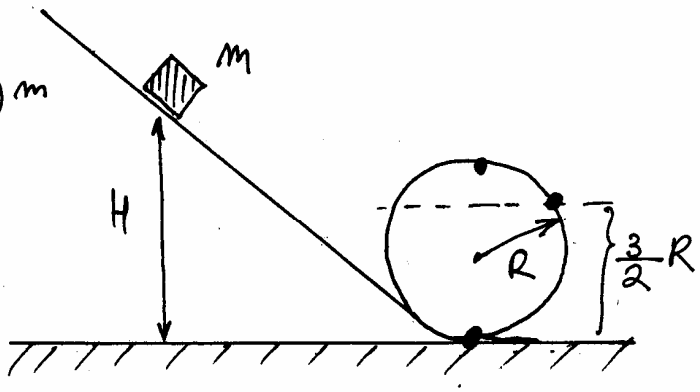

Fig. 2.

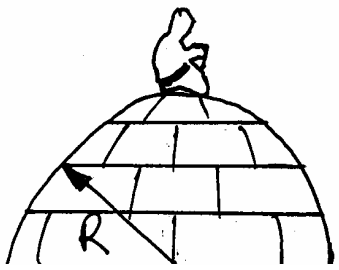

Fig.3.